Exercises 7 — propositional logic

1.

Find DNFs for the following formulae. Write “none” if a formula has no DNF.

1. $\neg((r \vee q) \leftrightarrow (q \vee p))$

   $(\neg p \wedge \neg q \wedge r) \vee (p \wedge \neg q \wedge \neg r)$

2. $\neg((p \to q) \vee (q \to r) \vee (r \to p))$

   none

3. $(p \to q) \wedge (q \to r) \wedge (r \to p)$

   $(p \wedge q \wedge r) \vee (\neg p \wedge \neg q \wedge \neg r)$

4. $(r \vee q) \to (q \vee p)$

   $p \vee q \vee (\neg q \wedge \neg r)$

5. $(p \to q) \to ((q \to r) \vee (r \to p))$

   $p \vee \neg q \vee r \vee \neg r \vee (p \wedge \neg q) = p \vee \neg q \vee r \vee \neg r = \text{true}$

6. $(p \to (q \wedge r)) \vee (q \to (p \wedge r)) \vee (r \to (p \wedge q))$

   $\neg p \vee \neg q \vee \neg r \vee (p \wedge q) \vee (p \wedge r) \vee (q \wedge r)$

7. $(r \vee q) \to (q \wedge p)$

   $(p \wedge q) \vee (\neg q \wedge \neg r)$

8. $(p \to q) \to ((q \to r) \wedge (r \to p))$

   $(p \wedge r) \vee (p \wedge \neg q) \vee (\neg q \wedge \neg r)$

2.

Let $E = p \to (q \to r)$ be a Boolean formula with three elementary letters, p, q, r.

1. Find an equivalent DNF for E.

   $\neg p \vee \neg q \vee r$

2. Find an equivalent *full* DNF for E. As a reminder, a DNF is full iff every letter (in this case, p, q, r, occurs in each of the basic conjunctions.

   $(p\wedge q\wedge r)\vee(p\wedge\neg q\wedge r)\vee(p\wedge\neg q\wedge\neg r)\vee(\neg p\wedge q\wedge r)\vee(\neg p\wedge q\wedge\neg r)\vee(\neg p\wedge\neg q\wedge r)\vee(\neg p\wedge\neg q\wedge\neg r)$

3. Find an equivalent form of E using only the $\barwedge$ connective. Remember, that it was defined as $\alpha \barwedge \beta = \neg(\alpha \wedge \beta)$, and that $\neg\alpha = \alpha \barwedge \alpha$, so it's probably a good idea to first find a form without disjunctions (or), and then go from there. Careful: The $\barwedge$ connective is NOT associative, i.e. it is NOT the case that $(\alpha \barwedge \beta) \barwedge \gamma$ is equivalent to $\alpha \barwedge (\beta \barwedge \gamma)$!

   $((p \barwedge q) \barwedge (p \barwedge q)) \barwedge (r \barwedge r)$

4. Fill in the truth table for E.

p	q	r	E
1	1	1	**1**
1	1	0	**0**
1	0	1	**1**
1	0	0	**1**
0	1	1	**1**
0	1	0	**1**
0	0	1	**1**
0	0	0	**1**

3.

Show that $\overline{\wedge}$ is not associative.

To show: there are p, q, r, such that $(p \,\overline{\wedge}\, q) \,\overline{\wedge}\, r \neq p \,\overline{\wedge}\, (q \,\overline{\wedge}\, r)$

$(0 \,\overline{\wedge}\, 0) \,\overline{\wedge}\, 1 = 1 \,\overline{\wedge}\, 1 = 0$

$0 \,\overline{\wedge}\, (0 \,\overline{\wedge}\, 1) = 0 \,\overline{\wedge}\, 1 = 1$