Exercises 8 — quantificational logic

1.

As we saw in the lecture on quantificational logic, $\exists x \forall y Rxy$ always implies $\forall x \exists y Rxy$ for any relation *R*. The converse, however, is not necessarily the case: $\forall x \exists y Rxy$ does not always mean that $\exists x \forall y Rxy$ is true.

Define a binary relation R over a non-empty universe D (that you also need to define) such that $\forall x \exists y Rxy$ is true, and $\exists x \forall y Rxy$ is false.

Hint: Keep in mind that the $\forall$ and $\exists$ operators are quantified over D.

D = ____________________________________

R = ____________________________________

2.

True or false?

1. $\forall x \in \mathbb{R}\ (x^2 > 0)$	true	false
2. $\forall x \in \mathbb{R}\ (\exists n \in \mathbb{N}\ (x^n \geq 0))$	true	false
3. $\exists a \in \mathbb{R}\ (\forall x \in \mathbb{R}\ (ax = x))$	true	false
4. $\forall X \in \mathcal{P}(\mathbb{N})\ (X \subseteq \mathbb{R})$	true	false
5. $\forall n \in \mathbb{N}\ (\exists X \in \mathcal{P}(\mathbb{N})\ (\#X \leq n))$	true	false
6. $\exists X \in \mathcal{P}(\mathbb{N})\ (\forall n \in \mathbb{N}\ (\#X \leq n))$	true	false
7. $\forall X \in \mathcal{P}(\mathbb{N})\ (\exists n \in \mathbb{Z}\ (\#X = n))$	true	false
8. $\forall n \in \mathbb{Z}\ (\exists X \in \mathcal{P}(\mathbb{N})\ (\#X = n))$	true	false
9. $\forall n \in \mathbb{N}\ (\exists X \in \mathcal{P}(\mathbb{N})\ (\#X = n))$	true	false
10. $\forall m \in \mathbb{Z}\ (\exists n \in \mathbb{Z}\ (m = n + 5))$	true	false
11. $\exists m \in \mathbb{Z}\ (\forall n \in \mathbb{Z}\ (m = n + 5))$	true	false
12. $\exists n \in \{k \in \mathbb{N} : k > 2\}\ (\exists a, b, c \in \mathbb{N}^+\ (a^n + b^n = c^n))$	true	false [1]
13. $\forall x, y \in \mathbb{R}\ (x < y \rightarrow \exists m \in \mathbb{R}\ (x < m < y))$	true	false
14. $\exists a, b, c \in \{0, 1\}\ (a \barwedge (b \barwedge c) = (a \barwedge b) \barwedge c)$	true	false
15. $\forall a, b, c \in \{0, 1\}\ (a \barwedge (b \barwedge c) = (a \barwedge b) \barwedge c)$	true	false
16. $\exists a \in \mathbb{R}\ (\forall x, y \in \mathbb{R}\ (ax = y))$	true	false
17. $\forall x, y \in \mathbb{R}\ (\exists a \in \mathbb{R}\ (ax = y))$	true	false

1 I would not ask this one in an exam, but perhaps you recognize it. If not, look up “Fermat’s last theorem”.