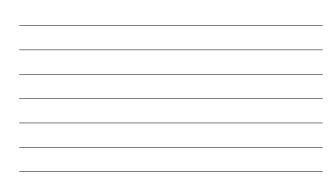






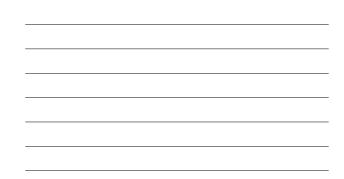
orall x(lpha) re	prese	la $\alpha$ that depends on a variable x : ents " <i>for all x,</i> $\alpha$ " ents " <i>there exists an x, such that</i> $\alpha$	" $ \begin{array}{c}                                     $	
Forall" is unive	rsal q	uantification, "exists" is existential.		218
		English	Symbols	
	l	All composer are poets	$\forall x (Cx \to Px)$	
Examples:	2	Some composers are poets	$\exists x(Cx \land Px)$	
			$\forall x(Px \rightarrow \neg Cx)$	
	3	No poets are composers	$\mathbf{v}_{\mathcal{X}}(P_{\mathcal{X}} \rightarrow \neg C_{\mathcal{X}})$	
$\forall x(\exists u(x \mid u))$	$-\frac{3}{4}$	No poets are composers Everybody loves someone	$ \begin{array}{c} \forall x(Px \rightarrow \neg Cx) \\ \forall x \exists y(Lxy) \end{array} $	
$\forall x(\exists y(x \bot y))$	3 4 5			
		Everybody loves someone	$\forall x \exists y (Lxy)$	
$\forall x \exists y (x \perp y)$	5	Everybody loves someone There is someone who is loved by everyone	$\forall x \exists y(Lxy)$ $\exists y \forall x(Lxy)$	
	5	Everybody loves someone There is someone who is loved by everyone There is a prime number less than 5 Behind every successful man stands an ambitious	$ \begin{aligned} &\forall x \exists y(Lxy) \\ &\exists y \forall x(Lxy) \\ &\exists x(Px \land (x < 5)) \end{aligned} $	
$\forall x \exists y (x \perp y)$	5 6 7	Everybody loves someone There is someone who is loved by everyone There is a prime number less than 5 Behind every successful man stands an ambitious woman	$ \begin{aligned} &\forall x \exists y(Lxy) \\ &\exists y \forall x(Lxy) \\ &\exists x(Px \land (x < S)) \\ &\forall x[(Mx \land Sx) \rightarrow \exists y(Wy \land Ay \land Byx)] \end{aligned} $	

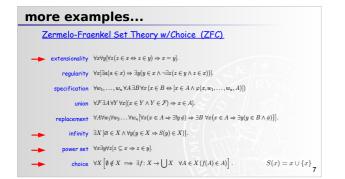


	ted in speaking about elements of some set. cified when the variable is introduced:
$\forall x \in A \; (lpha)$ represents "for	all x in A, $\alpha$ "
$\exists x \in A \; (lpha) \;\;$ represents "the	ere exists an x in A, such that $lpha$ "
This is just syntactic sugar:	
$\forall x \in A \ (\alpha) \dashv \vdash \forall x (x \in A \rightarrow$	α)
$\exists x \in A \ (\alpha) \dashv \exists x (x \in A \land c)$	) / ~/ ~ // 888
rue or false?	COLLENN
$\forall r \in \mathbb{R} \ (\exists n \in \mathbb{N} \ (n=r))$	$k n \leftrightarrow \; \exists a \in \mathbb{Z} \; (ak = n)$
$\exists n \in \mathbb{N} \ (\forall r \in \mathbb{R} \ (n=r))$	$n \in \mathbb{P} \leftrightarrow \dots n \in \mathbb{N}_2 \land \forall k \in \mathbb{N}_2 \ (k n \to k = n)$
$\forall n \in \mathbb{N} \ (\exists r \in \mathbb{R} \ (n = r))$	$\mathbb{N}_2 = \{n \in \mathbb{N}: n > 1\}$

more syntactic suga	r	
Often, one quantifier is used to introduce s	several variables:	
$\forall x, y, z \in A \ (\alpha) \text{ represents "forall } x$ ,	y, z in A, $\alpha$ "	
$\exists x, y, z \in A \ (\alpha)$ represents "there ex	rist x, y, z in A, such that $\alpha''$	
This, too, is just syntactic sugar:		
$\forall x, y, z \in A \ (\alpha) \dashv \!\!\! \dashv \!\!\! \forall x \in A \ (\forall y \in A)$	$(\forall z \in A \ (\alpha)))$	
$\exists x, y, z \in A \ (\alpha) \dashv \!$	$(\exists z \in A \ (lpha)))$	
True or false?		
$\forall n \in \mathbb{N} \; (\exists a, b \in \mathbb{N} \; (a < n < b))$	$R \subseteq A \times A \text{ transitive} \leftrightarrow \dots$	
$\forall a, b \in \mathbb{N} \ (\exists n \in \mathbb{N} \ (a < n < b))$	$\forall a, b, c \in A \ (aRb \wedge bRc \rightarrow aRc)$	
$\forall a, b \in \mathbb{N} \ (a < b \rightarrow \exists n \in \mathbb{N} \ (a < n < b))$	$\forall a,b \in A \ (aRb \rightarrow R(b) \subseteq R(a))$	
How can we "fix" this?	$orall (a,b) \in R \; (R(b) \subseteq R(a))$	5

Broad category	Specific items	Signs used	Purpose	$\forall a, b \in A \ (aRb \to R(b) \subseteq R(a))$
Basic terms	Constants	a, b, c,	Name-specific objects: for example, 5, Charlie Chaplin, London	$\forall k \in \mathbb{N}_2 \ (k n \to k = n)$
	Variables	<i>x</i> , <i>y</i> , <i>z</i> ,	Range over specific objects, combine with quantifiers to express generality	$\exists a \in \mathbb{Z} \ (ak = n)$
Function	2-Place	f, g, h,	Form compound terms out of	
letters als	so 1-place!		simpler terms, starting from the basic ones	N/~ IKVMAN
	n-Place operators,	too: +, - *, /,	ousie ones	
Predicates	1-Place	P, Q, R,	For example, is prime, is funny, is polluted	Yo` // @%\ 189
also other	2-Place		For example, is smaller than,	
relation	n-Place		resembles For example, lies between	formulae
symbols	/ini lace		(3-place)	
	Special relation sign	=	Identity	<u>8</u>
Quantifiers	Universal	¥	For all	TTTTP TO THE
	Existential	Э	There is	
Connectives	'Not' etc.	$\neg,\wedge,\vee,\rightarrow$	Usual truth tables	
Auxiliary	Parentheses and commas		To ensure unique	
			decomposition and make formulae easier to read	SLAM, p. 219 6





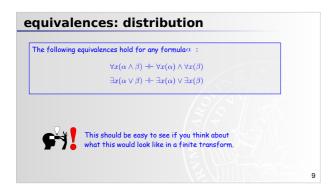
## finite transforms

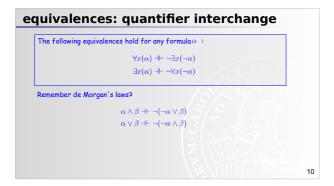
Suppose we quantify all variables over a finite set D, and we have constant symbols  $a_1,\ldots,a_n$  for each of its elements.

A *finite transform* of a universally/existentially quantified formula removes the quantifier, and instantiates the body for each element of D in a chained conjunction/disjunction.

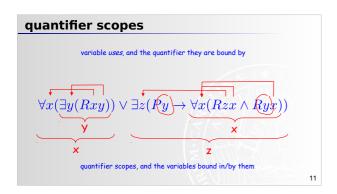
8

 $\exists x(Qx \to (Pxa \land Pxb)) \qquad (Qa \to (Paa \land Pab)) \lor (Qb \to (Pba \land Pbb))$ 

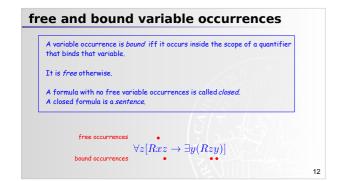


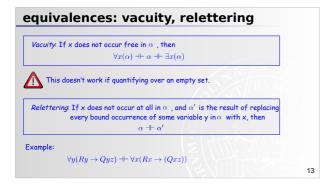


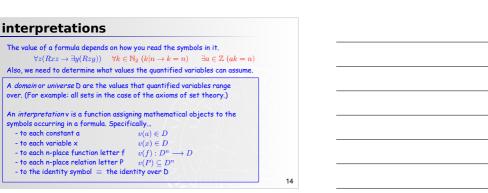












evaluating terms an	d formulae		
Given a domain D and an interpretation v, t	he value of a term t is	defined as follows:	
$\int v(a)$	for every constant $a$		
$v:t\mapsto \langle v(x)$	for every variable $\boldsymbol{x}$		
$v:t\mapsto \begin{cases} v(a)\\ v(x)\\ v(f)(v(t_1),,v(t_n)) \end{cases}$	for $t = f(t_1,, t_n)$		
With this, we can determine the truth valu	• • • • • • • • • • • •	as follows:	
$\int v(t_1) = v(t_2)$	for $\alpha = t_1 \equiv t_2$		
$(v(t_1,, v(t_n)) \in v(P)$	for $\alpha = Pt_1t_n$		
$\neg v(\beta)$	for $\alpha = \neg \beta$		
$v: \alpha \mapsto \begin{cases} v(t_1) = v(t_2) \\ (v(t_1,, v(t_n)) \in v(P) \\ \neg v(\beta) \\ v(\beta) \land v(\gamma) \end{cases}$	for $\alpha = \beta \wedge \gamma$		
?		SLAM 9.3,	
$\langle \langle \rangle \rangle$	for $\alpha = \forall x(\beta)$ for $\alpha = \exists x(\beta)$	pp. 227-233	1

A <i>logically implie</i> formulae in A arc	rmulae $A = \{\alpha_1,, a_n\}$ and a formula $\beta$ , we say that s $\beta$ iff <u>there is no interpretation v</u> such that all the : true under v, but $\beta$ is false:	
	$\beta \Longleftrightarrow \neg \exists v (\neg v(\beta) \land \forall \alpha (\alpha \in A \to v(\alpha)))$	
Also: $\alpha \dashv\vdash \beta \iff$	$\alpha \vdash \beta \land \beta \vdash \alpha$ logical equivalence	
$\emptyset \vdash \alpha$	logical truth	



